Case 1 TE (transverse Electric)

take xy as the plane of incidence

$$\overrightarrow{E_{is}} = \overrightarrow{E_{i\perp}}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$= \hat{y} E_{i\perp} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

The reflected and transmitted feilds.

$$\overrightarrow{E_{rs}} = \hat{y} E_{r\perp} e^{-j\beta_1(x\sin\theta_r + z\cos\theta_r)}$$

$$\overrightarrow{E_{ts}} = \hat{y} E_{t\perp} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\overrightarrow{H_{is}}(r_p) = \frac{1}{\eta_1}(\hat{x}\sin\theta_i + \hat{z}\cos\theta_i) \times \overrightarrow{E_{is}}$$

$$= \frac{1}{\eta_1} E_{i\perp} e^{-j\beta_1(x\sin\theta_i + \hat{z}\cos\theta_i)}$$

$$* (-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)$$

$$\overrightarrow{H_{rs}}(r_p) = \frac{1}{\eta_1}(\hat{x}\sin\theta_i - \hat{z}\cos\theta_i) \times \overrightarrow{E_{rs}}$$

$$= \frac{1}{\eta_2} E_{r\perp} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

$$* (+\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)$$

$$\overrightarrow{H_{ts}}(\vec{r}) = \frac{1}{\eta_2} E_{t\perp} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$* (-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t)$$

$$\overrightarrow{H_r}(\vec{r},t) = Re\left[\overrightarrow{H_{rs}} e^{j\omega t}\right]$$

At $z=0$ (interface)

$$E_{i\perp} + E_{r\perp} = E_{t\perp}$$

$$\frac{1}{\eta_1}(-\cos\theta_i E_{i\perp} + \cos\theta_i E_{r\perp}) = \frac{1}{\eta_2}(E_{t\perp}(-\cos\theta_t$$

$$\Gamma_\perp = \frac{E_{r\perp}}{E_{i\perp}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_2\cos\theta_t}$$

$$\tau_\perp = \frac{E_{t\perp}}{E_{i\perp}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

Note $\theta_i = 0$ and $\theta_t = 0$ then we have normal instance.

Parrallel case (for transverse magnetic)

$$\vec{E}_{is} = E_{i\parallel}\,\underbrace{(\hat{x}\cos\theta_i - \hat{z}\sin\theta_i)}_{\text{direction of } E_i}\,e^{-j\beta_1\overbrace{(x\sin\theta_i + z\cos\theta_i)}^{\hat{n}\cdot\vec{r}}}$$

$\hat{n} = \hat{x}\sin\theta_i + \hat{z}\cos\theta_i$

$$\vec{E}\cdot\hat{n} = 0$$

$$\vec{H}_{is} = \frac{1}{\eta_1}(\hat{x}\sin\theta_i + \hat{z}\cos\theta_i) \times \vec{E}_{is}$$

$$= \hat{y}\,\frac{1}{\eta_1}E_{i\parallel}\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{E}_{rs} = E_{r\parallel}(\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)\,e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

$$\vec{E}_{ts} = E_{t\parallel}(\hat{x}\cos\theta_t - \hat{z}\sin\theta_t)\,e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

Tangental component $\vec{E}$: $\hat{x}$

$$E_{i\parallel}\cos\theta_i + E_{r\parallel}\cos\theta_i = E_{t\parallel}\cos\theta_t$$

$$\frac{1}{\eta_1}(E_{i\parallel} - E_{r\parallel}) = \frac{1}{\eta_2}E_t$$

$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Note: equations are given, so make sure to know how to apply.

WAVE GENERATION

(radiation $\leftrightarrow$ antenna)

Blanket assumption for this course...
Far feild aproximation.

$$\therefore \quad \underbrace{K}_{\frac{2\pi}{d}} R_p \ggg 1 \qquad \therefore \quad R_p \ggg d$$

we are many wavelengths away.

$$\vec{H}(\vec{r_p}) = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A_s}$$

$$\oint \vec{B} \cdot d\vec{S} = 0 \quad \Rightarrow \vec{\nabla} \cdot \vec{B} = 0$$

$$\Rightarrow \vec{B} = \vec{\nabla} \times (\text{something})$$

$\overrightarrow{A_s}$ is determined by the corrent distributions
What of $\vec{E}$?

In electrostatic:

$$\oint \vec{E}\cdot d\vec{l} = 0 \Rightarrow \vec{E} = -\vec{\nabla} V$$

For time deperdante feilds:

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{s}$$

$$\vec{\nabla} \cdot \vec{\nabla}\times(\text{something})$$

$$\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$$

$$\vec{A}\cdot(\text{something}) = 0$$

In electrostatics:

$$\vec{E} = -\vec{\nabla} V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\int(\vec{\nabla}\times\vec{A})\cdot d\vec{s} = \int \vec{A}\cdot d\vec{l}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{A} \cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial}{\partial t} \vec{A}$$

$$\vec{H} = \frac{1}{\mu} \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\vec{\nabla} V \quad \text{(electro statics)}$$

$$V \longrightarrow V' = V + K$$

$$\vec{E} \longrightarrow \vec{E}' = -\vec{\nabla} V' = -\vec{\nabla} V = \vec{E}$$

adding a constant does not affect E.

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$A \rightarrow \vec{A}' = \vec{A} + \vec{\nabla} f$$

$$\vec{\nabla} \times A' = \vec{\nabla} \times \vec{A} + (\vec{\nabla} \times \vec{\nabla} f) = \vec{\nabla} \times \vec{A}$$

$$\boxed{\vec{\nabla} \cdot \vec{A}(\vec{R_p}, t) = -\mu_0 \varepsilon_0 \frac{\partial}{\partial t} V(\vec{R_p}, t)}$$